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Latent Curve Analysis

► Latent Growth Curve Modeling

Latent Growth Curve Modeling

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Synonyms

Latent curve analysis; Latent trajectory models; Structural equation models

Definition

A method for modeling repeated measures as latent variables is composed of a random intercept and random slope(s) that permit individual cases to have unique trajectories of change over time. Latent variables representing trajectories can be predicted from other variables and can be used to predict outcome variables in the model.

Description

Contemporary social scientists increasingly recognize the importance of collecting longitudinal data for understanding individual differences in stability and change. Traditional methods often rely on restrictive assumptions about covariance structures and complete case data that preclude research questions about individual differences in patterns of change over time. Developments in the areas of latent variable modeling and factor analysis formed the basis for *latent growth curve modeling* (Bollen & Curran, 2006; Curran et al., 2010; Duncan & Duncan 2004; Meredith & Tisak, 1990), a powerful and flexible technique for analyzing longitudinal data. The structure of a latent growth curve model is based on the assumption that an underlying, unobserved (i.e., *latent*) growth process is responsible for the pattern of change observed in repeated measures. Latent growth curve models allow formal tests of hypotheses about the mean rate of change, or *trajectory*, of an entire sample and explain individual differences in trajectories as a function of a set of covariates that are either time invariant (e.g., biological sex) or time varying (e.g., symptoms of depression).

Unconditional Growth Model

A latent growth curve model is *unconditional* when it excludes covariates that may explain individual differences in trajectories. As an illustrative example, consider a study that examines the trajectory of change in older adults' quality of life with three repeated measures (Zaninotto, Falaschetti, & Sacker, 2009). The equation for the unconditional model is

$$y_{it} = \alpha_i + \lambda_t \beta_i + \varepsilon_{it}$$

where y_{it} is the value of the quality of life variable for person i at time t, α_i is the random intercept for person i, λ_t represents the numerical value of Time at time t, β_i is the random slope for person i, and ε_{it} is the disturbance for person i at time t. By allowing the intercept and slope to vary randomly over individuals, the equations for the intercept and slope are as follows:

$$\alpha_i = \mu_\alpha + \zeta_{\alpha i}$$

$$\beta_i = \mu_\beta + \zeta_{\beta i}$$

where μ_α and μ_β are the mean intercept and mean slope across all cases (the *fixed effects*) and $\zeta_{\alpha i}$ and $\zeta_{\beta i}$ are disturbances that represent deviations of individual intercepts and slopes from the means across cases (the *random effects*). The unconditional model can represent the time trend as a single, linear slope, or it may include nonlinear forms of growth such as polynomial (e.g., quadratic, cubic) or piecewise trajectories. In the illustrative example, growth was a negative, linear slope, indicating that older adults' quality of life tends to decline at a constant rate over time.

Conditional Growth Models

The unconditional model can be expanded to include one or more *time-invariant covariates* (TIC) that explain how individuals differ from the intercept and slope for all cases. In the illustrative example, a time-invariant effect of sex on the intercept indicated that men reported lower quality of life on average compared to women. There was no effect of sex on the slope, however, indicating that the rate of decline in quality of life was similar for men and women.

The unconditional model can also be expanded to include one or more *time-varying covariates* (TVCs) that explain how individuals deviate from the mean trajectory at any given point in time. In the illustrative example, there was a time-varying effect of depressive symptoms, indicating that older adults who felt more depressed at any given point in time reported lower quality of life. Whereas time-invariant covariates can explain individual differences in rate of change compared to the mean trajectory, time-varying covariates serve to *shift* a person off their trajectory, explaining time-specific deviations in the underlying growth process.

Latent growth curve modeling is a flexible method for evaluating change in repeated measures that can accommodate multivariate designs, distal outcomes predicted by growth, and models in which intercepts and slopes vary across multiple groups, among other expansions, allowing analysts to adapt models to suit their research questions. Complex models, however, are more challenging to estimate and may require larger sample sizes and more waves of data to arrive at reliable solutions. They require careful consideration of the correspondence between the hypothesized and analytical models to avoid misleading results due to improper model specification. As longitudinal data collection becomes increasingly popular, carefully implemented latent growth curve models can enrich research efforts by expanding the range of questions it is possible to address.

Cross-References

▶ Latent Variable Path Models
▶ Latent Variables
▶ Longitudinal Data Analysis
▶ Longitudinal Structural Equation Modeling
▶ Measurement Error

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Latent Happiness Variable

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Definition

The latent happiness variable is the variable which is unobservable, but which is assumed to exist and which describes the happiness as it is distributed in the population under study.

Description

Happiness as measured in a sample and happiness in the population are two variables with fundamentally different distribution properties. While the former is observable by definition and has a discrete polytomous distribution, the latter is postulated to have a continuous distribution on a two-sided bounded interval as its domain; see entry ▶ Rescaling. The population happiness variable is unobservable or *latent* and is most important, because its characteristics are (to be) applied in correlational studies of happiness.

Properties of the Latent Happiness Variable

Kalmijn et al. (Kalmijn, 2010; Kalmijn, Arends, & Veenhoven, 2011) postulate a latent happiness variable H, to which the following properties are assigned:

1. H is postulated to be a variable, which is measured at the interval level of measurement and is expressed as a real number in the closed interval [0, 10].