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CHAPTER 9

DISAGGREGATING WITHIN-PERSON AND BETWEEN-PERSON EFFECTS IN MULTILEVEL AND STRUCTURAL EQUATION GROWTH MODELS

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Growth models are being used to study interindividual differences in intraindividual change at a rapidly increasing rate in the social sciences. These are most often estimated as either a structural equation model (SEM) or as a multilevel linear model (MLM), although other estimation methods are available (e.g., fixed effects growth, general estimating equations, etc.). Regardless of approach, the core concept behind a growth model is to use a set of repeated measures to infer the existence of one or more parameters that define an unobserved (or latent) trajectory over time. The functional form of the trajectories might be flat with respect to time (e.g., an intercept-only model), they might be linearly increasing or decreasing, or they might be some complex nonlinear function. Whatever the form, growth models typically estimate the fixed and random effects associated with stability and change over time. The fixed effects capture the mean of the trajectory pooling over all of the individuals within the sample and the random effects reflect individual variability around the mean trajectory. Smaller random effects suggest greater similarity in growth across individuals; larger random effects suggest greater individual heterogeneity in change over time.

It is common to include one or more exogenous predictors in a growth model. Measures that directly predict the growth trajectories are often called time-invariant covariates (or TICs) because these are believed to be unrelated to the passage of time. Examples of covariates that are truly time-invariant include biological sex, birth order, country of origin, race, and certain genetic markers. In principle, a truly time-invariant covariate can be assessed at any point in time given that the measure is independent of time. A related type of TIC is a measure that might be expected to vary as a function of time but only the initial assessment of the measure is included in the model. For example, although adolescent deviant peer affiliations might theoretically vary over time, an application might only consider a single measure of deviant peer affiliations taken at the initial assessment period (e.g., Chassin, Curran, Hussong, & Colder, 1996). Regardless of type, one or more TICs are used to predict variability in the parameters that define the trajectory over time.

Although not always explicitly recognized in many substantive applications, the inclusion of TICs as predictors of growth provide for direct tests of *between-person* differences in growth. These between-person differences are captured in the estimated regression parameters that reflect a shift in the conditional means of the distribution of trajectory parameters as a function of a shift in the mean of the TICs. For example, boys might report significantly higher initial values relative to girls, or individuals in a treatment condition might report significantly steeper increases in an outcome over time relative to individuals in a control condition. TICs are thus person-specific measures that are constant over time and capture between-person effects by directly predicting the trajectory parameters.

However, there is another important component of individual change that has received much less attention in the estimation of many growth models, and this is the systematic study of *within-person* influences on change over time. Whereas the estimation of between-person differences is based on measures taken at a single point in time (i.e., TICs), the estimation of within-person differences involves measures taken repeatedly over time; as such, these measures are referred to as time-varying covariates, or TVCs. Unlike TICs (which directly influence the random growth parameters), the TVCs directly influence the repeated assessments of the outcome measure *net* the influence of underlying growth in the outcome. Whereas the between-person TIC effect reflects a mean shift in the parameters defining growth pooling across the sample of individuals, the within-person TVC effect reflects a mean shift in the time-specific outcome measure as a function of the relative distribution of the TVCs *within* each individual. More colloquially, TICs provide insights regarding "for whom" an effect exists and TVCs provide insights regarding "at what time" an effect exists. TVCs are thus person- and time-specific measures that capture within-person effects by directly predicting the repeated measures above and beyond systematic growth in the outcome over time.

There is an added complexity to the within-person and between-person distinction that is better known in the quantitative literature but is much less evident in substantive research applications. Specifically, it is possible that a person- and time-specific TVC simultaneously exerts *both* a within-person and a between-person effect. The within-person effect is captured in the relation between the person- and time-specific measure of the TVC and the person- and time-specific measure of the outcome; the between-person effect is captured in the relation between the person-specific (and thus time invariant) *mean* of the TVC and the growth trajectories. Either confounding or misattributing these two levels of effects can lead to potentially significant errors of inference (see Curran & Bauer, 2011, for a recent review). Only by simultaneously considering these two types of influences can the full nature of the relation between a TVC and the outcome be understood (Raudenbush & Bryk, 2002; Schwartz & Stone, 1998; Singer & Willett, 2003).

As a reflection of the potentially complex nature of these relations, a TVC might exert a between-person effect but not a within-person effect. For example, Hoffman and Stawski (2009) studied the relation between negative mood, stress, and physical symptoms in a sample of younger and older adults. One key finding was that there was support for a significant positive between-person effect between daily negative mood and daily physical symptoms, but there was no support for a within-person effect between the same two measures (Hoffman & Stawski, 2009, Figure 2). In other words, individuals reporting greater overall negative mood tended to report higher overall physical symptomatology; however, there was no relation between a time-specific elevation of negative mood relative to the individual's overall baseline in the prediction of an associated time-specific elevation of physical symptoms.

Alternatively, a TVC might exert a within-person effect but not a between-person effect. For example, Rodebaugh, Curran, and Chambless (2002) studied the relation between daily anxiety and panic expectancy. Individuals reporting higher levels of expectancy of panic relative to their overall baseline in the evening were more likely to report experiencing a panic attack the following day (Rodebaugh et al., 2002, Figure 2). However, there was no support for a meaningful between-person effect on the same measures. In other words, individuals reporting elevations in expectancy relative to their norm on one day tended to report higher incidents of panic the following day; however, there was not a systematic relation between the overall person-specific levels of expectancies and person-specific panic pooling over all repeated measures across individuals.

It is thus possible that a TVC exerts only a between-person effect, only a within-person effect, neither, or both. Examining only one level of influence can meaningfully alter substantive conclusions about the true structural relation between the TVC and the outcome. One might miss an effect that actually exists, or one might mistakenly obtain an aggregate effect that reflects neither the between-person nor within-person relation. It is thus critical that these influences be appropriately disaggregated, evaluated, and interpreted.

However, as we describe in detail later, the MLM and SEM approaches to growth modeling incorporate and evaluate these TVC effects in quite different ways. Indeed, what initially appear to be identically parameterized models result in very different estimates of between-person and within-person effects. These differences may in turn have a significant impact on how TVC-relevant hypotheses are tested and interpreted in practice. As we demonstrate later, different conclusions could be drawn about the nature of a relation depending on whether an SEM or MLM approach was fitted to the same measures. Despite the potential importance, we believe these issues have not been fully explicated, either analytically or substantively. Our motivating goal here is to both conceptually and statistically compare how TVC effects are estimated within the MLM and SEM growth modeling traditions, particularly with respect to the disaggregation of the within- and between-person effects of the TVC on the outcome.

We begin with a review of the standard growth model estimated within the MLM framework and demonstrate that both the within- and between-person effects of the TVC can be directly obtained using well-established methods, at least when certain underlying assumptions are met. Next, we review the standard growth model estimated within the SEM framework and demonstrate that only the within-person effect of a TVC can be directly obtained. We then show that the same methods used to disaggregate effects in the MLM cannot be applied within the SEM and we analytically explicate precisely how the SEM and MLM approaches incorporate TVCs in quite different ways. We then explore several options for obtaining disaggregated estimates within the SEM and we propose a new approach that disattenuates the estimate of the between-person effect for sampling variability in the person-mean. We conclude with current limitations of our approach and offer recommendations for the use of these techniques in applied research.

THE MULTILEVEL GROWTH MODEL

The growth model within the MLM framework is motivated by the fact that multiple repeated measures are nested within individual, and this results in a natural two-level hierarchical data structure (e.g., Bryk & Raudenbush, 1987; Raudenbush & Bryk, 2002; Singer & Willett, 2003). As we will see, although there are many similarities between the SEM and MLM growth models, there are several critically important points of divergence (e.g., Bauer, 2003; Curran, 2003; Raudenbush, 2001; Willett & Sayer, 1994). Given the many differences in the parameterization of the same model within the SEM and MLM, we use a notation scheme that is consistent with the MLM tradition.

To begin, we define y_{ti} to represent an observed repeated measure on construct y at time point t for individual i. For a linear growth model, the observed repeated measure can be expressed as a function of an individually varying intercept and slope weighted by time. This is given as

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{ti} + e_{ti} \tag{9.1}$$

where β_{0i} and β_{1i} represent the intercept and linear slope for individual i, respectively; $time_{ti}$ is the observed value of time at assessment t for individual i; and e_{ti} is the time- and individual-specific residual. Note that for comparison to SEMs we describe later, time enters the MLM as a numeric level-1 predictor variable (i.e., $time_{ti}$).

An important characteristic of the growth model is that the intercept and slope values are treated as random variables. In other words, these are governed by a bivariate probability distribution and can thus be expressed as

$$\begin{aligned} \beta_{0i} &= \gamma_{00} + u_{0i} \\ \beta_{1i} &= \gamma_{10} + u_{1i} \end{aligned} \tag{9.2}$$

where γ_{00} and γ_{10} are the overall mean intercept and slope, respectively, and u_{0i} and u_{1i} are the individual-specific deviations from these means, respectively. Finally, equation 9.2 can be substituted into equation 9.1 to define the reduced form expression of the model:

$$y_{ti} = \left(\gamma_{00} + \gamma_{10} time_{ti}\right) + \left(u_{0i} + u_{1i} time_{ti} + e_{ti}\right) \tag{9.3}$$

where the first parenthetical term reflects the fixed effects for the model, and the second parenthetical term reflects the random effects and all is defined as before.

The parameters that define the MLM described in equations 9.1 and 9.2 are $E(\beta_{0i}) = \gamma_{00}$, $E(\beta_{1i}) = \gamma_{10}$, $\text{var}(u_{0i}) = \tau_{00}$, $\text{var}(u_{1i}) = \tau_{11}$, and $\text{var}(e_{ti}) = \sigma_t^2$. The covariance between random effects is commonly estimated as part of this model as well (e.g., $\text{cov}\left[u_{0i}, u_{1i}\right] = \tau_{10}$). Finally, although there are a number of alternative possible structures for σ_t^2, here we assume the residuals are independent and homoscedastic over time (i.e., $\sigma_t^2 = \sigma^2$ for all t). This is a simplifying restriction that does not impact any of our later developments.

This model can be expanded to include one or more time-invariant covariates (TICs), and these enter into the level-2 equations. For a single TIC, denoted w_i, equation 9.2 would be expanded so that

$$\begin{aligned} \beta_{0i} &= \gamma_{00} + \gamma_{01} w_i + u_{0i} \\ \beta_{1i} &= \gamma_{10} + \gamma_{11} w_i + u_{1i} \end{aligned} \tag{9.4}$$

where γ_{01} and γ_{11} represent the fixed-effect regression of the intercept and slope factors on the TIC, respectively. It is clear from the subscripting that w_i is a time-invariant covariate given that the value is unique to individual i but does not vary as a function of time point t. As such, the regression coefficients associated with the TIC are explicit estimates of the between-person effects.

Additionally, one or more TVCs can be incorporated into the level-1 equation. Whereas the TICs enter into the level-2 equations, the TVCs enter into the level-1 equations. For example, for a single TVC denoted z_{ti}, the level-1 equation is given as

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{ti} + \beta_{2i} z_{ti} + e_{ti} \tag{9.5}$$

where z_{ti} represents the time-varying covariate z at time t for individual i, and all else is defined as above.

Although the influence of the TVC (i.e., β_{2i}) can be defined as random (Raudenbush & Bryk, 2002, equation 6.21), we focus our discussion here on the TVC defined as having only a fixed effect; this implies that the magnitude of the relation between the TVC and the outcome is constant across individuals. We do this to allow for more direct comparisons between the MLM and the SEM, the latter of which does not easily allow for the estimation of random effects for the TVCs.[1] The level-2 equations are thus

$$\begin{aligned} \beta_{0i} &= \gamma_{00} + u_{0i} \\ \beta_{1i} &= \gamma_{10} + u_{1i} \\ \beta_{2i} &= \gamma_{20} \end{aligned} \tag{9.6}$$

with reduced form

$$y_{ti} = \left(\gamma_{00} + \gamma_{10} time_{ti} + \gamma_{20} z_{ti}\right) + \left(u_{0i} + u_{1i} time_{ti} + e_{ti}\right). \tag{9.7}$$

As before, the first parenthetical term contains the fixed effects and the second contains the random effects. Importantly, the regression of the outcome on the TVC (i.e., γ_{20}) captures the shift in the conditional mean of y_{ti} per unit shift in z_{ti} net the effect of the underlying trajectory. However, as we will see next, this is an aggregate effect that inextricably combines the within-and between-person components of the relation between the TVC and the outcome. We must take additional steps to disaggregate these two influences, and it is to this we now turn.

DISAGGREGATING BETWEEN- AND WITHIN-PERSON EFFECTS

It is well known that, under certain assumptions, within- and between-person effects can be efficiently and unambiguously disaggregated within the MLM using the strategy of person-mean centering (e.g., Hofmann & Gavin, 1998; Kreft, de Leeuw, & Aiken, 1995; Raudenbush & Bryk, 2002, p. 183). Traditionally, the term *centering* is typically used to describe the rescaling of a random variable by deviating the observed values around the mean of the variable (e.g., Aiken & West, 1991, pp. 28–48). For example, within the standard fixed-effects regression model, a predictor x_i is centered via $x_i' = x_i - \bar{x}$, where $\bar{x}$ is the observed mean of x_i and x_i' is the mean-deviated rescaling of x_i (see, e.g., Cohen, Cohen, West, & Aiken, 2003, p. 261). By definition, the mean of a centered variable is equal to zero, and this property offers both interpretational and sometimes computational advantages in a number of modeling applications.

However, centering becomes more complex when considering TVCs. This is because multiple repeated measures are nested within each individual, and there are thus two means to consider: the grand mean of the TVC pooling over all time points and all individuals, and each person-specific mean pooling over all time points *within* individuals. To better see this, reconsider equation 9.7, in which we defined the TVC to be z_{ti}. There are two ways that we can center the TVC.

First, we can deviate the TVC around the grand mean pooling over all individuals. Here, $\ddot{z}_{ti} = z_{ti} - \bar{z}$, where $\ddot{z}_{ti}$ represents the grand-mean-centered TVC, z_{ti} is the observed TVC, and $\bar{z}$ is the grand mean of z_{ti} pooling over all individuals and all time points. Second, we can deviate the TVC around the person-specific mean of the TVC unique to each individual. Specifically, $\dot{z}_{ti} = z_{ti} - \bar{z}_i$, where $\dot{z}_{ti}$ represents the person-mean-centered TVC, z_{ti} is again the observed TVC, and $\bar{z}_i$ is the person-specific mean for individual i. Either z_{ti}, $\dot{z}_{ti}$, or $\ddot{z}_{ti}$ can be used as the level-1 predictor in equation 9.7. Importantly, all three scalings of the TVC will result in precisely the same model fit (i.e., in terms of deviance and all deviance-based measures); however, the associated regression parameters and variance components offer different interpretations, sometimes markedly so (see, e.g., Raudenbush & Bryk, 2002, Table 5.10).

Although the between- and within-person effects can be disaggregated using any of the three scalings of the TVC, these effects can be most efficiently obtained within the multilevel model using the person-mean-centered TVC at level 1 (i.e., $\dot{z}_{ti}$) and the person-specific mean at level 2 (i.e., $\bar{z}_i$). The level-1 equation is given as

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{ti} + \beta_{2i}\dot{z}_{ti} + e_{ti} \quad (9.8)$$

the level-2 equation as

$$\begin{aligned} \beta_{0i} &= \gamma_{00} + \gamma_{01}\bar{z}_i + u_{0i} \\ \beta_{1i} &= \gamma_{10} + \gamma_{11}\bar{z}_i + u_{1i} \\ \beta_{2i} &= \gamma_{20} \end{aligned} \quad (9.9)$$

and the reduced-form equation as

$$y_{ti} = \left(\gamma_{00} + \gamma_{10} time_{ti} + \gamma_{20}\dot{z}_{ti} + \gamma_{01}\bar{z}_i + \gamma_{11}\bar{z}_i time_{ti}\right) + \left(u_{0i} + u_{1i} time_{ti} + e_{ti}\right) \quad (9.10)$$

where γ_{00} is the intercept, γ_{10} is the fixed effect of time, γ_{20} is a direct estimate of the within-person effect, γ_{01} is a direct estimate of the between-person effect, and γ_{11} is the cross-level interaction between the person-specific mean and time.[2] As before, the within-person and between-person effects are net the contribution of the linear effect of time.

Although these methods for disaggregating effects are well established and widely used, there are also several assumptions that must hold for these techniques to work properly; we explore these assumptions in detail in Curran and Bauer (2011). Of key interest to our discussion here is the assumption that the within-person variability among the set of repeated measures of the TVCs is equal to zero; in other words, it is assumed that the person-specific mean is assessed with perfect reliability. This can most clearly be seen in that the person-specific mean of the TVCs is computed and used as a level-2 predictor, but the within-person standard deviation of the TVCs around the person-specific mean is discarded. Omitting this important source of within-person sampling variability leads to bias in the estimation of the true variance of the person-specific mean and in turn attenuates the estimate of the between-person effect; see Lüdtke et al. (2008) for an excellent recent discussion of this issue.

As a concrete example, consider two individuals who have precisely the same mean value among a set of TVCs, say $\bar{z}_1 = \bar{z}_2 = 5.00$. This reflects that the overall level of the TVC is equal for these two individuals and each obtains the same value for the analysis. However, say that the standard deviations for the two individuals were $sd_1 = .50$ and $sd_2 = 2.50$, respectively; these standard deviations capture the within-person variability of the repeated measures around the person-specific mean. The difference in magnitude of these standard deviations reflects that the repeated assessments of the TVC are much more tightly clustered around the mean for the first individual compared to that of the second. Yet when using the person-mean as a manifest predictor of the random intercept of y_{ti}, the between-person differences in within-person variability are not introduced into the model; indeed, both are assumed to be zero and these two individuals would be treated as identical with respect to the TVC.

Within both the MLM and SEM, exogenous covariates are assumed to be fixed and known and thus error free. Although within-person sampling variability among the TVCs is not measurement error in the traditional psychometric sense of the term (e.g., as in measurement error in classical test theory), the MLM and SEM assume this source of variability to be zero. It is well known within the general linear model that violating the assumption of perfectly reliable exogenous covariates tends to attenuate the associated regression coefficient (see Bollen, 1989, pp. 151–175, and Lüdtke et al., 2008). With respect to our work here, we expect that omitting within-person variability would drive down the sample estimate of the between-person effect, potentially significantly so (Lüdtke et al., 2008, Figure 1). Given this, we pay particular attention to the potential attenuating effects of omitting within-person variability when estimating the between-person effect in both the MLM and SEM.

To demonstrate the use of existing methods for disaggregating effects within the MLM and to subsequently compare these to the SEM, we next use computer simulation methodology to fit the model defined in equation 9.10 to artificially generated data with known population structure.

SIMULATED DATA

We begin our evaluation of the MLM using computer simulation methodology that allows us to draw unambiguous conclusions about the relative accuracy of parameter estimation given the known population structure. We drew upon recent published applications of growth models along with our collective experience to define a population model that we felt was both realistic and constructed in a way as to highlight the differences between within-person and between-person effects of the TVC. We based our population model on a hypothetical relation between anxiety symptoms and alcohol use. The positive between-person effect reflects that, on average, individuals who are characterized by higher overall levels of anxiety tend to drink at higher levels (e.g., possibly due to drinking alcohol to lessen anxiety symptoms; Kassel et al., 2010). The negative within-person effect reflects that, on average, individuals tend to drink less when anxiety symptoms are elevated relative to their own baseline (e.g., when anxious they avoid social situations where alcohol is available; Kaplow, Curran, & Costello, 2001). Thus, whereas individuals who are more anxious tend to drink more (the positive between-person effect), individuals tend to drink less on days where they are experiencing higher anxiety (the negative within-person effect).[3]

Consistent with this hypothetical example we defined the population-generating model to be a growth model with both a random intercept and a random positive linear slope of time as well as main effects of the within- and between-person TVC.[4] Of key interest to us here are the fixed effects of the within- and between-person effects of the TVC on the outcome. The population value of the within-person effect was set to –.25, indicating that higher values of the time-specific TVC are associated with lower values on the outcome. The population value of the between-person effect was set to .75, indicating that higher values of the person-specific overall level of the TVC are associated with higher values on the outcome. We generated data in SAS Version 9.2 based on $T = 5$ repeated measures assessed on $N = 250$ individuals and we created $R = 1{,}000$ data replications.[5] All tabled values present the means and standard deviations of the parameter estimates pooling over the 1,000 replications.

Fitting the MLM

We fitted a number of different models to precisely the same artificial data that in turn allows us to directly compare the MLM to the SEMs that we will define in a moment. We began by fitting a standard linear growth model without the inclusion of any other predictors (i.e., equation 9.3); these results are labeled *Model 1* in the first column of Table 9.1. The mean intercept over the 1,000 replications was 22.496 ($SD = .127$) and the mean slope was 1.998 ($SD = .071$). Thus the model-implied starting point was approximately 22.5 and this increased at approximately two units per unit-time.

We then expanded this growth model to include just the main effect of the uncentered TVC (i.e., z_{ti}); more specifically, we fitted the model

$$y_{ti} = (\gamma_{00} + \gamma_{10} time_{ti} + \gamma_{20} z_{ti}) + (u_{0i} + u_{1i} time_{ti} + e_{ti}) \tag{9.11}$$

TABLE 9.1 Means and Standard Deviations of Parameter Estimates for Multilevel Model Summarized over 1,000 Replications at Sample Size $N = 250$

Parameter	Model 1 Growth only	Model 2 Growth plus uncentered TVC	Model 3 Growth plus centered TVC	Model 4 Growth plus centered TVC plus person-mean
Intercept ($\hat{\gamma}_{00}$)	22.496 (.127)	22.262 (.368)	22.495 (.126)	17.841 (1.047)
Time ($\hat{\gamma}_{10}$)	1.998 (.071)	1.999 (.070)	1.999 (.070)	2.020 (.589)
TVC ($\hat{\gamma}_{20}$)	—	–.177 (.034)	–.250 (.035)	–.250 (.035)
Mean ($\hat{\gamma}_{01}$)	—	—	—	.465 (.103)
Interaction ($\hat{\gamma}_{11}$)	—	—	—	–.002 (.059)
$\hat{\tau}$	2.537 (.351)	2.832 (.375)	2.564 (.346)	2.248 (.318)
$\hat{\tau}_{01}$	–.197 (.143)	–.198 (.147)	–.197 (.141)	–.193 (.134)
$\hat{\tau}_{11}$	1.009 (.108)	1.009 (.107)	1.009 (.107)	1.004 (.107)
$\hat{\sigma}^2$	2.128 (.110)	2.011 (.103)	2.000 (.102)	2.000 (.102)

Note: First number is mean and number in parentheses is standard deviation; dashes indicate that the corresponding value was not estimated as part of the associated model.

to the 1,000 simulated samples of size $n = 250$. The model results are presented under *Model 2* in Table 9.1. Most important to our discussion here, the mean of the 1,000 sample estimates of the main effect of the uncentered TVC was equal to –.177 ($SD = .034$). As expected, because in this model the TVC is in the raw scale metric, this effect represents the *aggregate* relation between the TVC and the outcome. Given that the population within-person effect is –.25 and the population between-person effect is .75, this aggregate effect reflects neither the within-person nor between-person effects (Raudenbush & Bryk, 2002, equation 5.38). However, we can use the methods described above to disaggregate these two levels of effect.

To accomplish this we fitted the same model as above to the same 1,000 simulated samples but this time we used the person-mean-centered scaling of the TVC (i.e., $\dot{z}_{ti}$); these results are summarized under *Model 3* in Table 9.1. Again, most important to our discussion here, the mean of the 1,000 sample estimates of the main effect of the person-mean-centered TVC was equal to –.25 ($SD = .037$). Given that we are using the person-mean-centered scaling of the TVC, this represents a direct estimate of the within-person effect; indeed, the obtained mean of the sample estimates is precisely equal to the population-generating value. However, we still have no information regarding the between-person effect; we can incorporate the person-mean as a level-2 predictor to obtain this effect.

To obtain estimates of both the within-person and between-person effects, we expanded the prior model to include the person-specific mean as a level-2 predictor of the random intercept.[6] Thus the model fitted to the 1,000 simulated samples corresponds to equation 9.10; the results are summarized under *Model 4* in Table 9.1. The mean of the 1,000 within-person estimates was –.25 ($SD = .035$) and of the between-person estimates was .465 ($SD = .103$). Note that whereas the within-person effect is accurate with respect to the population model, the between-person effect is substantially attenuated relative to the corresponding population value (see Lüdtke et al., 2008, for a clear description of why this attenuation occurs). Given that the population value is .75 and obtained value is .47, the sample estimate of the between-person effect is underestimated by nearly 40% within the MLM.

This brings us to our first observation:

As expected from existing analytic theory, the within- and between-person effects can be simultaneously and unambiguously disaggregated within the MLM by incorporating the person-centered TVC as a level-1 predictor and the person-mean as a level-2 predictor; however, the between-person effect is attenuated due to the omission of within-person variability around the person-specific mean.

To clarify, we have yet to offer any new developments thus far and have primarily reviewed and demonstrated existing knowledge in this area. However, we now turn to a closer examination of how these same effects are obtained within the SEM, a topic that in our opinion is much less well understood.

THE STRUCTURAL EQUATION GROWTH MODEL

The growth model is conceptualized in the MLM framework as modeling a hierarchical data structure that is induced by the nesting of multiple repeated observations within each individual. In contrast, the growth model is conceptualized in the SEM framework as modeling multiple observed repeated measures as manifest indicators that define an underlying latent growth process (e.g., Bollen & Curran, 2006; McArdle, 1988, 1989, 1991; McArdle & Epstein, 1987; Meredith & Tisak, 1984, 1990). Switching notation to stay consistent with the SEM framework, we begin by considering a linear growth model in which the measurement equation is defined as

$$y_{ti} = \alpha_i + \lambda_{ti}\beta_i + \varepsilon_{ti} \qquad (9.12)$$

where y_{ti} is the observed outcome for individual i at time point t, α_i and β_i are the person-specific intercept and slope components for the linear trajectory for individual i, λ_{ti} is the numerical value of time at time point t, and ε_{ti} is the residual term for individual i at time point t. Because we conceptualize the intercept and slope components as random variates (e.g., realizations randomly drawn from a bivariate probability distribution), we can write structural equations for these terms as

$$\alpha_i = \mu_\alpha + \zeta_{\alpha_i}$$
$$\beta_i = \mu_\beta + \zeta_{\beta_i} \qquad (9.13)$$

where μ_α and μ_β are the mean intercept and slope components, respectively, and ζ_{α_i} and ζ_{β_i} are the individual-specific deviations around these mean values for individual i. Finally, the structural equation can be substituted into the measurement equation to produce the reduced form equation as

$$y_{ti} = \left(\mu_\alpha + \lambda_{ti}\mu_\beta\right) + \left(\zeta_{\alpha_i} + \lambda_{ti}\zeta_{\beta_i} + \varepsilon_{ti}\right) \qquad (9.14)$$

where all terms are defined as above.

Because we are working within the framework of the general SEM, the time-specific measures of the outcome y_{ti} are treated as observed manifest

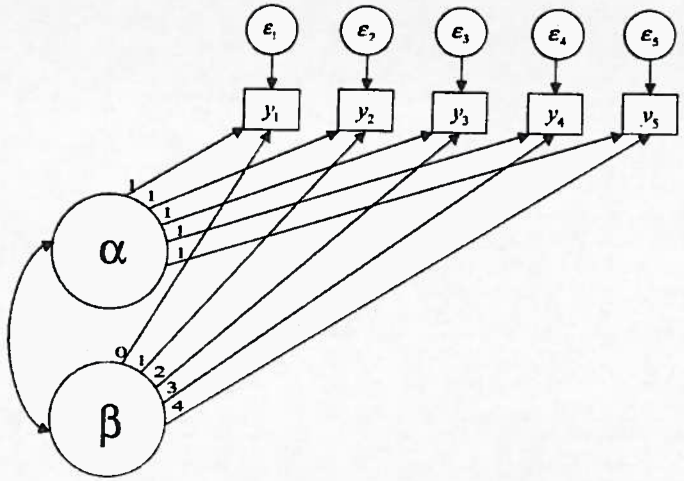

Figure 9.1 Path diagram for a five-time-point unconditional linear SEM growth model.

variables. As such, each trajectory component is defined as an unobserved latent variable with a mean (i.e., the fixed effect) and a variance (i.e., the random effect). Finally, the numerical values of time are fixed factor loadings that relate the observed variables to the latent variables. A path diagram for this model is presented in Figure 9.1.

This linear growth model is defined by two fixed effects (i.e., $E(\alpha_i) = \mu_\alpha$ and $E(\beta_i) = \mu_\beta$) and three random effects (i.e., $\text{var}(\zeta_{\alpha_i}) = \psi_{\alpha\alpha}$, $\text{var}(\zeta_{\beta_i}) = \psi_{\beta\beta}$, and $\text{var}(\varepsilon_{ti}) = \sigma_t^2$). The covariance between the two random effects (i.e., $\text{cov}(\zeta_{\alpha_i}, \zeta_{\beta_i}) = \psi_{\alpha\beta}$) is typically estimated as part of the model. Furthermore, the time-specific residual variance (i.e., σ_t^2) can either vary as a function of time t or can be held constant over time; to remain consistent with the MLM defined earlier, we retain the assumption of homoscedasticity (i.e., $\sigma_t^2 = \sigma^2$ for all t).

The SEM can easily incorporate one or more predictors either within the measurement equation, the structural equations, or both. TICs are denoted by w_i and enter directly into the structural equations

$$\alpha_i = \mu_\alpha + \gamma_\alpha w_i + \zeta_{\alpha_i}$$
$$\beta_i = \mu_\beta + \gamma_\beta w_i + \zeta_{\beta_i} \quad (9.15)$$

where γ_α and γ_β represent the fixed effect of the TIC on the random intercepts and slopes, respectively. These fixed effects capture between-person

differences in the prediction of within-person change as a function of w_i. However, the focus of our work here is on the estimation and interpretation of effects of the TVCs.

We continue to denote the TVCs as z_{ti} to reflect that the obtained value varies as a function of time point t for individual i; this is in contrast to the TIC where w_i reflected that the obtained value only varies as a function i but not t. We can thus incorporate a TVC directly into the measurement equation as

$$y_{ti} = \alpha_i + \lambda_{ti}\beta_i + \gamma_t z_{ti} + \varepsilon_{ti} \quad (9.16)$$

where z_{ti} is the TVC for individual i at time point t, and γ_t is the fixed effect of the TVC on the outcome at time point t. The SEM allows the value of γ_t to take on unique values at each time point t; here we make the assumption that γ is constant over t to better make direct comparisons back to the MLM, although this in no way limits our later developments.[7] Because the TVC does not vary over individual, the structural equations remain defined as in equation 9.13, and the resulting reduced-form equation is

$$y_{ti} = (\mu_\alpha + \lambda_{ti}\mu_\beta + \gamma z_{ti}) + (\zeta_{\alpha_i} + \lambda_{ti}\zeta_{\beta_i} + \varepsilon_{ti}) \quad (9.17)$$

where all terms are defined as before. A path diagram of this model is presented in Figure 9.2. We now briefly demonstrate this model using the 1,000 samples of artificially generated data.

Fitting the SEM

We fit a series of SEMs to the same simulated data as we used earlier. We begin by fitting an unconditional growth model (i.e., the model presented in Figure 9.1) and, as expected, we obtain precisely the same parameter estimates as those obtained for the equivalently parameterized multilevel growth models. These results are summarized under *Model 1* in Table 9.2. We next extended this model to include the main effect of the uncentered TVC.

Descriptively, this is a random intercept and random linear slope growth model with z_{ti} as the single time-varying covariate, the magnitude of which is held constant over time. Consistent with the structure of the population model, we restricted the time-specific residual variance to be homoscedastic over time (i.e., $\text{var}(\varepsilon_{ti}) = \sigma^2$). Furthermore, consistent with the standard parameterization of SEMs, all exogenous variables were allowed to freely covary (i.e., all TVCs covaried with one another and with the random intercept). Finally, we used standard normal theory maximum likelihood estimation to obtain parameter estimates and standard errors.

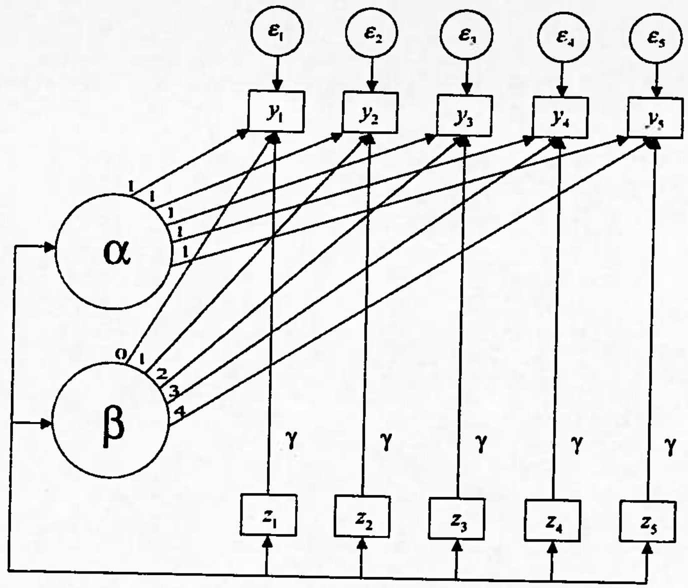

Figure 9.2 Path diagram for the SEM parameterization of the TVC model. All factor loadings and single-headed arrows from residuals are fixed to 1.0; single-headed arrows from *z* to *y* are regression coefficients; connected double-headed arrows represent all possible covariances.

The model results are summarized under *Model 2* in Table 9.2. The mean of the 1,000 sample estimates of the uncentered TVC was equal to –.25 (SD = .037). This is identical to the population within-person effect of –.25 and the sample point estimate is precisely equal to those obtained from the MLM using the person-specific mean-centered TVCs (compare *Model 3* in Table 9.1 with *Model 2* in Table 9.2). Quite interestingly, in direct contrast to the MLM, the inclusion of the TVC kept in its original scale (i.e., z_{ti}) in the measurement equation of the SEM accurately captured the *within-person* relation between the measure of z_{ti} and the outcome y_{ti}. Furthermore, note that there is no information available about the potential *between-person* relation between the TVC and the outcome. Recall that the structure of the artificial data includes a large and positive between-person relation (i.e., .75), yet no evidence of this between-person effect is obtained from these SEM results. Given our knowledge of the population-generating model, this is a salient omission from the SEM.

TABLE 9.2 Means and Standard Deviations of Parameter Estimates and Model Fit Statistics for Structural Equation Model Summarized over 1,000 Replications at Sample Size N = 250

	Model 1 Growth only	Model 2 Growth plus uncentered TVC	Model 3 Growth plus uncentered TVC and zero covs	Model 4 Growth plus latent TVC factor	Population-generating values
Intercept ($\hat{\mu}_\alpha$)	22.496 (.127)	24.992 (.391)	24.263 (.368)	14.901 (1.553)	15.0
Time ($\hat{\mu}_\beta$)	1.998 (.071)	1.999 (.070)	1.999 (.070)	2.029 (.829)	2.0
$\hat{\gamma}$	—	–.250 (.037)	–.177 (.034)	–.250 (.035)	–.25
$\hat{\gamma}_\alpha$	—	—	—	.759 (.154)	.75
$\hat{\gamma}_\beta$	—	—	—	–.003 (.083)	0
$\hat{\delta}_0$	—	.999 (.153)	—	—	1.0
$\hat{\delta}_1$	—	–.003 (.082)	—	—	0
$\hat{\psi}_{\alpha\alpha}$	2.537 (.351)	2.987 (.389)	2.832 (.375)	1.952 (.332)	2.0
$\hat{\psi}_{\alpha\beta}$	–.197 (.143)	–.199 (.149)	–.198 (.147)	–.191 (.135)	–.20
$\hat{\psi}_{\beta\beta}$	1.009 (.108)	1.010 (.107)	1.009 (.107)	1.003 (.107)	1.0
$\hat{\sigma}^2$	2.128 (.110)	2.000 (.102)	2.011 (.103)	2.000 (.102)	2.0
χ^2	13.718 (5.145)	28.146 (7.437)	286.717 (38.823)	53.253 (10.367)	53
df	14	28	48	53	53

Note: First number is mean and number in parentheses is standard deviation; dashes indicate that the corresponding value was not estimated as part of the associated model.

This brings us to our second observation:

The inclusion of an uncentered time-varying covariate in a standard SEM provides an accurate estimate of the within-person effect of the TVC on the outcome, but no information is available about the potential between-person effect of the TVC.

We view this as an important observation for two reasons. First, we conducted an extensive literature review of TVCs within the SEM and did not find a single instance in which a TVC effect was unequivocally interpreted as a within-person effect. This includes our own work on this same topic. For example, Bollen and Curran (2006) present results from an SEM growth model in which math ability is the dependent measure and reading ability is the TVC. The relation between the TVC and the outcome is described as "...a significant and positive prediction of math ability from the contemporaneous influence of reading ability ($\hat{\gamma} = .41$, $p < .0001$)" (p. 197). This is not an incorrect interpretation, but it is an imprecise interpretation. As a within-person effect, the more precise interpretation of the TVC is that higher reading scores *relative to the individual's baseline reading skills* are associated with higher math scores *relative to the individual's baseline math skills.* It is thus not an aggregate interpretation (e.g., overall reading predicting overall math), but a more precise relation of time-specific standing relative to a person-specific baseline. Given the lack of attention currently paid to the distinction of within-person and between-person effects within the SEM, it is likely that TVC effects within the SEM are not commonly interpreted in as precise and accurate a way as is otherwise possible.

A second concern that is highlighted by our artificial data results is that, despite the existence of a large and positive between-person effect that exists in the population model, there is no evidence of this effect in the SEM sample results. As such, a significant characteristic of the data structure is not captured in this model and this could in turn lead to limited or even misleading conclusions about the processes under study. Returning to the example presented in Bollen and Curran (2006, p. 197), no comment is made regarding potential between-person differences in the relation between reading and math ability; an aggregate interpretation was provided regarding the TVC effect and nothing more was said. This is a universal interpretation of TVC models estimated within the SEM, and a major component of the over-time relation between the TVC and the outcome is being ignored.

Returning to our simulated data, we have obtained a direct estimate of the within-person effect in the SEM via the uncentered TVC, but we have no information about the corresponding between-person effect. Given that the SEM and MLM are being fitted to precisely the same data, it seems logical that the well-established strategies used to disaggregate between- and within-person effects in the MLM could equivalently be applied to the SEM to obtain these same estimates. However, two critical problems are encountered when attempting to do this.

To begin, even though we demonstrated that the within-effect can be obtained in the SEM using the raw scale TVC (i.e., z_{ti}), we consider an SEM in which the person-mean-centered TVC is used instead (i.e., $\dot{z}_{ti}$). We do this to directly compare the SEM and MLM results when using the same scaling for the TVC.[8] However, we immediately encounter a significant problem: the standard SEM is not estimable under maximum likelihood (ML) estimation when using the person-centered TVC. The reason is that, within the framework of the SEM, the person-centered TVC is an *ipsative* measure (Cattell, 1944; Clemans, 1966). An ipsative measure is defined as one in which the sum of a set of items for a given individual is equal across all individuals within the sample. Because by definition $\Sigma\dot{z}_{ti} = 0$ over all t within i, $\dot{z}_{ti}$ is ipsative.

There is a long history in the development and application of statistical methods to ipsative measures (e.g., Cattell, 1944; Chan & Bentler, 1996; Dunlap & Cornwell, 1994). However, there is a particular characteristic of ipsative measures that is salient to our discussion here. Namely, the elements within any row or column of the covariance matrix among the set of ipsative items must sum to zero. As such, the matrix is singular and has a zero determinant. This poses a fatal problem for standard ML estimation within the SEM given the requirement that the log of the determinant of the sample covariance matrix be calculated during optimization (e.g., Bollen, 1989, equation 4.67). Although there are potential *ad hoc* ways to address this problem (e.g., ridge estimation, unweighted least squares), these are generally not acceptable given that these estimators can be biased and none result in true ML estimates of model parameters (e.g., Greene, 2000).

This leads us to our third observation:

The SEM is not estimable under standard maximum likelihood when using person-mean-centered TVCs because the TVCs are ipsative and the sample covariance matrix is singular.

To be clear, although we are not able to include the person-mean-centered TVC within the SEM, this is not problematic from a practical standpoint since we can obtain direct estimates of the within-person effect using the raw scaled TVCs. However, the ipsative nature of the person-mean-centered TVC highlights a distinct difference in how the TVCs are being incorporated into the MLM and SEM. More importantly, for the moment we remain unable to obtain a direct estimate of the between-person effect within the SEM.[9] Continuing to draw on the established strategies used in the MLM, we next consider incorporating the person-mean as a predictor to obtain these between-person effects.

Between-Person Effects within the SEM

We continue to follow the recommended strategy for use within the MLM to disaggregate between- and within-person effects; that is, we can extend the SEM expression for the TVC model to include the person-mean as a predictor of the random intercept. The measurement equation remains unchanged

$$y_{ti} = \alpha_i + \lambda_{ti}\beta_i + \gamma z_{ti} + \varepsilon_{ti} \quad (9.18)$$

but the structural equations are now

$$\alpha_i = \mu_\alpha + \gamma_\alpha \bar{z}_i + \zeta_{\alpha_i}$$
$$\beta_i = \mu_\beta + \gamma_\beta \bar{z}_i + \zeta_{\beta_i} \quad (9.19)$$

with reduced form

$$y_{ti} = (\mu_\alpha + \lambda_{ti}\mu_\beta + \gamma z_{ti} + \gamma_\alpha \bar{z}_i + \gamma_\beta \lambda_{ti} \bar{z}_i) + (\zeta_{\alpha_i} + \lambda_{ti}\zeta_{\beta_i} + \varepsilon_{ti}) \quad (9.20)$$

where $\bar{z}_i$ is the person-specific mean of the TVC for individual i pooling over all time points t, γ_α and γ_β are the between-person effects of the TVC on the outcome, and all else is as defined before.

Despite the importance of including the person-mean as a predictor within the MLM, this same model defined in equation 9.20 is not estimable within the SEM framework. There is a remarkably simple reason: the covariance matrix of the TVCs (i.e., z_{ti}) and the person-specific mean ($\bar{z}_i$) is singular, again making optimization impossible under ML estimation. Interestingly, this indeterminacy has a markedly different source than the one we previously encountered. Whereas the prior indeterminacy arose from the ipsative nature of the person-mean-centered TVCs (i.e., $\dot{z}_{ti}$), here the singularity is due to the fact that there is a direct linear dependency between the set of uncentered scores z_{ti} and the person-mean $\bar{z}_i$. Indeed, this is precisely the example we commonly use when teaching introductory statistics as to why the degrees-of-freedom for computing the sample variance is $n - 1$: namely, that knowledge of the sample mean restricts one dimension of variability in the data. As such, there is a column dependency, the determinant of the sample covariance matrix is zero, and ML estimation is again not possible.

This brings us to our fourth observation:

It is not possible to obtain an estimate of the between-person effect in the SEM by using the person-mean as an exogenous predictor because of the linear dependency between the person-mean and the TVCs in the covariance matrix.

Given this observation, there is not a method for simultaneously estimating the within-person effect and the between-person effect using the person-mean within the SEM. This is in direct contrast to the established strategies that are widely used within the MLM and leaves us with two logical questions. First, exactly what accounts for the differences between the MLM and SEM approaches to the TVC model? And second, is there a way to estimate the between-person effect within the SEM that does not rely on the inclusion of the person-mean as an exogenous predictor? We address each of these issues in turn.

THE SOURCE OF THE DISCREPANCY

The source of the discrepancy between the MLM and SEM estimates of the TVC effect relates to precisely how the two modeling approaches parameterize the covariance structure among the TVCs and the random intercept. In the Appendix we present the analytic derivations that explicate the relation between the SEM and MLM growth modeling frameworks. Here we augment these derivations with a more descriptive discussion of the two underlying models.

Consider the reduced-form expressions for a random intercept model with one TVC for the MLM

$$y_{ti} = (\gamma_{00} + \gamma_{10} time_{ti} + \gamma_{20} z_{ti}) + (u_{0i} + time_{ti} u_{1i} + e_{ti}) \quad (9.21)$$

and for the SEM

$$y_{ti} = (\mu_\alpha + \lambda_{ti}\mu_\beta + \gamma z_{ti}) + (\zeta_{\alpha_i} + \lambda_{ti}\zeta_{\beta_i} + \varepsilon_{ti}). \quad (9.22)$$

Despite the apparent structural similarities between these two equations, fundamentally different assumptions underlie each expression. In the MLM it is explicitly assumed that the predictors (i.e., z_{ti}) are uncorrelated with the random effects (u_{0i} and e_{ti}). More formally, $\text{cov}(z_{ti}, u_{0i}) = 0$ for all t and all i. Although an explicit assumption of the model (e.g., Raudenbush & Bryk, 2002, p. 255, point #6), in a very real sense these covariances do not even exist in the MLM; that is, these parameters are not a formal part of the model, which is why assumptions are required that these covariances equal zero in the population.

In contrast, in the SEM these same covariances are an explicit part of the model. More formally, $\text{cov}(z_{ti}, \zeta_{\alpha_i}) = \psi_{z_t,\alpha}$ for $t = 1, 2, \ldots, T$. In other words, the model allows for the estimation of covariances between the random intercept and all time-specific TVCs (this can be seen in the double-headed arrows between the TVCs and the intercept factor in the path diagram presented in Figure 9.2). Furthermore, these covariances can be freely estimated or fixed to zero in any given application. Thus, in the MLM the covariances between the TVCs and the random intercept are assumed to be zero; in the SEM these covariances may be freely estimated as part of the growth model. This is precisely where the difference in the estimation of the between- and within-person effects lies.

Let us first consider the MLM. As described in Raudenbush and Bryk (2002, p. 256, point #6), the assumption that predictors at one level are uncorrelated with random effects at the other level implies that biases are not introduced by the omission of relevant predictors at either level. However, if the uncentered TVC (i.e., z_{ti}) is used as a level-1 predictor, and no other predictors are included at level 2, there will be a nonzero covariance induced between the TVCs and the random intercept (assuming an intraclass correlation for the TVCs that is greater than zero; see Raudenbush & Bryk, 2002).

The reason is that the uncentered measure of the TVC contains information about *both* within-person differences *and* between-person differences. Specifically, the TVC can be expressed as

$$z_{ti} = \bar{z}_i + \varepsilon_{z_{ti}} \tag{9.23}$$

where the first term varies *between* individuals (i.e., the person-specific mean $\bar{z}_i$) and the second term varies *within* individuals (i.e., the person-specific and time-specific residual $\varepsilon_{z_{ti}}$). When using the raw-scaled TVC there is thus a between-person component embedded within the TVCs that has been omitted from the model, and this in turn induces a cross-level correlation, a correlation that the MLM assumes to be zero.

However, this model misspecification can be circumvented in one of two ways. First, the TVC can be person-mean centered, thus making the within- and between-person effects orthogonal and meeting the assumption of zero covariance. Second, the person-specific mean can be included as a level-2 covariate, thus incorporating the previously omitted predictor and correcting for the bias (Raudenbush & Bryk, 2002, pp. 261–262). Both of these approaches are often used in many MLM applications.

In contrast, in the SEM the covariance structure between the TVCs and the random intercept enters directly into the model formulation. As such, this allows for the random intercept effect to be estimated *net* the TVC effects and, most important to our discussion here, the TVC effects are *net* the influence of the random intercept. As such, the uncentered TVCs capture the within-person effect because these effects are unique to the influence of the random intercept. That is, the saturated covariance structure among the TVCs and the random intercept "absorbs" the omitted predictor, precisely as is possible in the general linear model. This is why all exogenous predictors freely covary in the multiple regression model (e.g., Greene, 2000).

To summarize, because the MLM assumes the covariance between the TVC and the random intercept to be zero, modifications must be made either directly to the data (via centering) or to the model (via inclusion of the person-mean) to address this issue. In contrast, because the SEM can explicitly incorporate these same covariances as part of the model, these cross-level relations can be directly modeled and the data need not be manually modified via data management outside of the analysis.

To briefly demonstrate the impact of the exogenous covariance structure, we reestimated the SEM using the uncentered TVC but we fixed the covariances between the TVCs and the random intercept to be zero (thus corresponding to the standard MLM parameterization). Consistent with expectations, the resulting TVC effect represented precisely the same aggregate effect as the MLM estimated to the uncentered TVC earlier; specifically, compare *Model 2* in Table 9.1 with *Model 3* in Table 9.2. Thus in the SEM the uncentered TVC results in a direct estimate of the *within*-person effect when the TVCs freely covary with the random intercept but results in an estimate of the *aggregate* effect when these covariances are held to zero.

To better explicate the critical role the exogenous covariances play in the SEM, we can capitalize on the information that is contained in the covariance structure between the TVCs and the random intercept and slope within the SEM to derive an estimate of the between-person effect. Consider again the reduced form of the SEM described above

$$y_{ti} = (\mu_\alpha + \lambda_{ti}\mu_\beta + \gamma z_{ti}) + (\zeta_{\alpha_i} + \lambda_{ti}\zeta_{\beta_i} + \varepsilon_{ti}) \tag{9.24}$$

in which γ represents a direct estimate of the within-person effect. Furthermore, define δ_0 to be the difference between the within- and between-person effects for the random intercept[10]; this is consistent with the compositional effect of Raudenbush and Bryk (2002, equation 5.42). Because the compositional effect is the difference between the between-person effect and the within-person effect, we can obtain an estimate of the between-person effect based solely on the within-person effect and the compositional effect; in other words, the between-person effect is the sum of the within-person effect and the compositional effect.

As we detail in the Appendix, a sample estimate of the compositional effect can be derived based solely on parameter estimates drawn from the

SEM TVC model defined in equation 9.24. Specifically, the compositional effect is given as

$$\delta_0 = \frac{\frac{1}{T}\sum_{t=1}^{T} \mathrm{cov}(z_{ti}, \zeta^*_{\alpha i})}{\frac{1}{T^2}\left\{\sum_{t=1}^{T} \mathrm{var}(z_{ti}) + \sum_{t \neq t'}^{T} \mathrm{cov}\left(z_{ti}, z'_{t'i}\right)\right\}} \tag{9.25}$$

where $t = 1, 2, \ldots, T$, $t \neq t'$, $z \neq z'$, and all else is defined as above. In other words, the estimate δ_0 is the ratio of the mean of the covariances between each TVC and the random intercept (the numerator) to the scaled summation of the variance of each TVC and the covariance of each TVC with all other TVCs (the denominator). Although the compositional effect is of substantive interest in some applications (e.g., Raudenbush & Bryk, 2002, pp. 139–141), here we only use this to obtain the between-person effect. More specifically, when δ_0 is added to the within-person effect, we obtain a direct estimate of the between-person effect even though we have no explicit information about between-person variability in the model (i.e., we have no information about $\bar{z}_i$ anywhere in the model).

To demonstrate the information contained in δ_0, we applied equation 9.25 to the results obtained from our earlier TVC model fitted to the 1,000 replications of sample size 250 within the SEM. Recall that our within-person effect was –.25. Applying equation 9.25 to the results from this SEM, we obtain a mean estimate for the compositional effect of $\hat{\delta}_0 = 1.0$. Thus our between-person effect is obtained as $\hat{\delta}_0 + \hat{\gamma} = 1.0 + (-.25) = .75$, which is precisely equal to the population between-person effect.

This brings us to our fifth observation:

Within the SEM, sufficient information is contained in the covariance structure among the TVCs and the random intercept to allow for the estimation of the between-person effect without requiring the inclusion of the person-specific mean as an exogenous predictor; furthermore, the between-person estimate is not attenuated given that the calculation does not rely on the person-specific mean.

This observation highlights exactly why the within-person effect is obtained from the uncentered TVC in the SEM but the aggregate effect is obtained from the uncentered TVC in the MLM. We can next consider what options the SEM might provide to move beyond the standard methods of disaggregation currently used in the MLM. Although the MLM is of course characterized by myriad significant strengths, embedding the TVC model within the SEM allows for a number of model expansions that are not currently available within the MLM (just as the MLM offers a number of model expansions that are not currently available within the SEM). Although there are a variety of interesting ways in which these models can be expanded within the SEM, here we focus on one important example: the estimation of the person-specific mean of the TVCs using a latent variable methodology to explicitly incorporate information about within-person sampling variability. It is to this we now turn.

ESTIMATING THE PERSON-MEAN VIA A LATENT FACTOR

We earlier demonstrated why the person-specific mean cannot be included in the SEM given the linear dependency between the person-mean and the TVCs. In contrast, the person-mean can be incorporated into the MLM to provide a direct estimate of the between-person effect of the TVC on the outcome. However, it is important to keep in mind that the person-mean that is used to predict the random intercept in the MLM is an exogenous manifest measure and, as such, is assumed to be error free (e.g., Bollen, 1989; Raudenbush & Bryk, 2002). The assumption of perfect reliability is widely known to be often dubious, the violation of which can yield biased parameter estimates and standard errors (e.g., Lüdtke et al., 2008). However, one of the key strengths of the SEM is that multiple indicator latent factors can be used to estimate the true score variance associated with a set of measures that in turn avoids the strong assumption of error-free predictors. We conclude by exploring how a latent variable approach can be used to explicitly include information about within-person sampling variability in the disaggregation of within- and between-person effects.

Earlier we described the potential importance of incorporating information about within-person variability when computing the person-specific mean of the TVC. We can draw on the developments of Rogosa and Saner (1995) to formalize these issues. We can express the TVC z_{ti} as an additive function of a person-specific mean and a time-specific deviation from that mean such that

$$z_{ti} = \bar{z}_i + \varepsilon_{z_{ti}} \tag{9.26}$$

where $\bar{z}_i$ is the mean for person i and $\varepsilon_{z_{ti}}$ is the time-specific deviation around this mean. The standard approach within the MLM is to use the person-specific mean as a level-2 predictor but to discard $\varepsilon_{z_{ti}}$. However, additional information is embedded in this term, specifically the variance of these time-specific deviations from the person-specific mean. This is simply given as $\mathrm{var}(\varepsilon_{z_{ti}}) = \hat{\sigma}^2_{z_i}$ and represents the within-person variance of the TVC over time.

Given the above, the grand mean of the set of N person-specific means is

$$\bar{\bar{z}} = \frac{\sum_{i=1}^{N} \bar{z}_i}{N} \tag{9.27}$$

and the grand mean of the set of N person-specific variances is

$$\hat{\sigma}_z^2 = \frac{\sum_{i=1}^{N} \hat{\sigma}_{z_i}^2}{N}. \tag{9.28}$$

We can now modify Rogosa and Saner's (1995) expressions for this simple case to define the true score variance of the sample means as

$$\hat{\psi}_{\alpha_z\alpha_z} = \mathrm{var}(\bar{z}_i) - \frac{\hat{\sigma}_z^2}{T} \tag{9.29}$$

where $\hat{\psi}_{\alpha_z\alpha_z}$ represents the estimate of true score variance, $\mathrm{var}(\bar{z}_i)$ represents the variance of the individual person-specific means, $\hat{\sigma}_z^2$ represents the mean of the individual person-specific variances, and T represents the total number of repeated assessments.[11]

Equation 9.29 demonstrates that the variance of the person-specific means is overestimated by an amount equal to $\hat{\sigma}_z^2 / T$. Thus, using the sample mean as a predictor of the random intercept implicitly (and unrealistically) assumes that $\hat{\sigma}_z^2 / T = 0$ for all i; that is, there is no error of estimation, and each within-person time-specific measure of the TVC is equal to the person-specific mean of the set TVCs.

However, we can simultaneously estimate both the true score variance and the residual variance in a straightforward manner within the SEM. Just as we earlier defined a random intercept factor for the outcome y_{ti}, we define a second random intercept factor for the TVC z_{ti}. We now write the reduced-form equation for the TVCs as

$$z_{ti} = \mu_{\alpha_z} + \zeta_{\alpha_{z_i}} + \varepsilon_{z_{ti}} \tag{9.30}$$

where $E(z_{ti}) = \mu_{\alpha_z}$, $\mathrm{var}(z_{ti}) = \mathrm{var}(\zeta_{\alpha_{z_i}} + \varepsilon_{z_{ti}}) = \psi_{\alpha_z\alpha_z} + \sigma_z^2$. We have now included a subscript z to distinguish these values from similar expressions for y in equation 9.17. Importantly, note the estimation of two variance components, one estimating true score variance ($\psi_{\alpha_z\alpha_z}$) and the other within-person residual variance (σ_z^2). We are thus able to disaggregate these two variance components associated with the TVC by defining a latent factor intercept for the TVCs.

We will use the random intercept latent factor for the TVC as defined in equation 9.30 as a predictor of the random intercept latent factor of the outcome y in order to incorporate this variance component into the model. Although this is quite straightforward (and is really nothing more than a second random intercept in a multivariate growth model), we encounter one final complexity that we must address. Because our goal is to obtain *simultaneous* estimates of the between- and within-person effects, we must retain our regression of outcome y_{ti} on the TVC z_{ti}. However, because we have defined a random intercept for z_{ti}, we must not regress y_{ti} directly on z_{ti} as we have done thus far in the SEM. Doing so will result in an *aggregate* effect of the TVC on the outcome.[12] Instead, we must estimate a nonstandard effect in which we regress y_{ti} on the time-specific *residual* of z_{ti}, specifically $\varepsilon_{z_{ti}}$. The motivation for this is that the time-specific residuals represent the deviation of each observation from the true mean of the TVC (e.g., see equation 9.30). We are thus mean-deviating each time-specific TVC through the parameterization of the latent factor as opposed to manually mean-deviating the observed scores as is done in the MLM. This parameterization allows for the simultaneous estimation of both the within- and between-person effects based solely on the inclusion of the raw-scaled TVC.

We present this final model in diagrammatic form in Figure 9.3.[13] We fitted this model to our artificially generated 1,000 datasets of $N = 250$ and obtained a mean within-person effect of –.25 ($SD = .035$) and a mean between-person effect of .759 ($SD = .154$); see Model 4 in Table 9.2 for com-

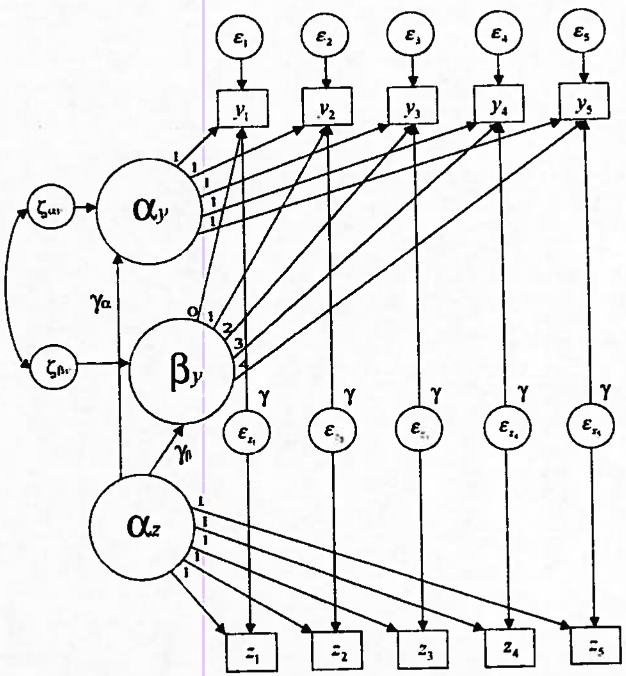

Figure 9.3 Final structural equation model for disaggregating within-person (denoted γ_1) and between-person (denoted γ_2) effects of the TVC on the criterion.

plete results. Importantly, note that the between effect is captured in the regression of the intercept factor for the outcome on the intercept factor for the TVCs. The within-person effect remains unchanged from before; this result was expected given that we are simply person-mean centering the TVCs through the parameterization of the model as opposed to manually calculating these outside of the analysis. However, the between-person effect is almost identical to the population-generating parameter that considers within-person variability around the person-specific mean. Furthermore, this obtained estimate is nearly 65% larger than that obtained from the MLM estimate of the within-person effect obtained that ignore within-person variability. This finding is directly consistent with expectations given that sampling variability in a predictor tends to negatively bias associated regression coefficients (Lüdtke et al., 2008). Because we have disaggregated true score variance and sampling variance, the between-person effect has been subsequently disattenuated for sampling variability and is thus a more accurate estimate of the corresponding population effect.

This brings us to our sixth and final observation:

The within-person and between-person effects of the TVC can be simultaneously disaggregated through the alternative parameterization of the SEM, and the between-person effect has been disattenuated by correcting for within-person sampling variability in the estimation of the person-mean.

CONCLUSION

We have explored a large number of topics with the goal of highlighting five specific issues related to the disaggregation of within-person and between-person effects in TVC growth models.

1. A multilevel model estimated with an uncentered TVC and no person-specific mean at level 2 will result in an aggregation of between- and within-person effects. Inclusion of the person-specific mean centered TVC at level 1 and the person-mean of the TVC at level 2 will result in direct estimates of the within- and between person effects, respectively. However, the between-person effect is attenuated due to the violated assumption of perfect reliability of the person-specific mean.
2. A standard SEM estimated under ML with a person-specific mean-centered TVC is not estimable due to the singularity of the covariance matrix, regardless of whether the person-specific means are included. Thus neither a within-person nor between-person effect can be obtained from inclusion of the person-specific mean-centered TVC in the SEM.
3. A standard SEM estimated with an uncentered TVC and no person-specific mean will result in a direct estimate of the within-person effect. Inclusion of the person-specific mean as a predictor of the intercept factor yields a model that is not estimable due to the singularity of the covariance matrix. Thus no between-person effect can be directly obtained from the SEM that only includes the uncentered TVC.
4. The between-person effect can be analytically computed based on the weighted combination of parameter estimates obtained from the standard SEM estimated with the uncentered TVC. This between-person effect is not attenuated given that the calculation does not rely on the inclusion of the person-specific mean.
5. An SEM estimated with an uncentered TVC and (a) a latent intercept factor estimated for the TVC, (b) the latent intercept of the outcome regressed on the latent intercept of the TVC, and (c) the repeated measures of the outcome regressed directly on the residuals of the TVC will result in direct estimates of both the between- and within-person effects. These estimates are unbiased and the between-person effect has been disattenuated for within-person sampling variability around the person-specific mean.

We have presented no novel developments in terms of the traditional methods used to disaggregate within- and between-person effects within the multilevel model. However, we believe that much less attention has been paid to the attenuating effects of estimating the between-person effect using the person-specific mean related to the omission of information about within-person variability among the set of TVCs (but see Lüdtke et al., 2008, for a discussion of the related topic of individuals nested within groups). More importantly, we are aware of no prior discussions of the many issues that are encountered when attempting to disaggregate effects within the SEM. Indeed, we were unable to identify a single published instance in which a TVC effect within the SEM was interpreted explicitly in terms of within-person effects; nor were we able to identify any prior discussions of estimating a between-person effect of a TVC within the SEM. This includes our own joint collection of work on this topic. As such, much potentially important information is omitted from these models and results are routinely imprecisely interpreted.

Yet, as we demonstrated above, obtaining a direct estimate of the between-person effect in the SEM is not a trivial issue. We cannot use a person-centered TVC because a singularity arises from the ipsative nature of this measure. And we cannot include the person-specific mean as a predictor because a singularity arises from the inclusion of the mean with the TVCs in the same covariance matrix.[14] However, we can obtain these effects through the direct parameterization of the model in which a random intercept is estimated for the TVCs themselves. This not only allows a method to obtain a direct estimate of the between-person effect, but has the added advantage of disaggregating true score variance from within-person sampling variance and is obtained under a true maximum likelihood estimator. Furthermore, once embedded within the SEM framework, many other modeling extensions are possible.

UNRESOLVED ISSUES AND DIRECTIONS FOR FUTURE RESEARCH

Although we believe we have been able to draw a number of general and novel conclusions about the disaggregation of within- and between-person effects in both the MLM and SEM, there are several unresolved issues to bear in mind. First, as we noted earlier, all of our above developments assume that there is not systematic growth in the TVCs over time (Curran & Bauer, 2011). This requirement holds for both the multilevel model using the person-mean at level 2 and the SEM using a latent factor for the TVCs. The reason for this requirement is clear: Deviating the TVCs around the person-mean assumes that the TVCs are not changing systematically over time (e.g., Curran & Bauer, 2011, Figure 9). If some form of a time trend underlies the TVCs, then the person-mean deviated scores that omit this trend are inappropriate. If this situation exists, more complex multivariate models are needed (e.g., Curran & Bollen, 2001; du Toit & Browne, 2001; MacCallum, Kim, Malarkey, & Kiecolt-Glaser, 1997; McArdle & Hamagami, 2001). Further work is needed to better understand precisely how the between- and within-person effects are being manifested within these more complex multivariate models.

Second, we have explored the issues of disaggregation only with respect to continuously distributed TVCs. However, normal distributions need not be a requirement for disaggregation using our proposed methods; assuming the TVCs approximate a continuous distribution, there are several well-developed estimation methods that circumvent the traditional assumption of multivariate normality (or, more precisely, no excessive kurtosis; Browne, 1984) invoked by normal theory ML (e.g., Satorra, 1990). Furthermore, no additional issues arise if the criterion measure is discretely scaled, and all of our developments would generalize to nonlinear link functions and alternative response distributions. However, several intriguing issues arise when the TVC itself is discretely scaled, and particularly when it is dichotomous. For example, the person-mean of the dichotomous TVC represents the proportion of total assessments in which the item was endorsed, and the person-mean-centered TVC would rescale the 0 or 1 around this person-mean. Furthermore, fitting a latent factor to a set of dichotomous indicators requires moving to a nonlinear SEM, thus resulting in a much more complicated model (e.g., Flora & Curran, 2004; Mehta, Neale, & Flay, 2004). Indeed, we typically cannot directly access time-specific residuals in an SEM with discrete dependent measures, yet this is a key component of our proposed latent variable model for the TVC (e.g., Figure 9.3). As such, our general conclusions are primarily restricted to the case of continuously (though not necessarily normally) distributed TVCs. There are thus a number of promising areas for future research both in terms of more precise substantive predictions about the exact nature of individual change as well as the development of new statistical models to capture such change.

It has long been known that there are a large number of advantages to the collection and analysis of repeated-measures data over time. Advantages include increased power, greater ability to study the psychometric properties of measures, and the direct examination of interindividual differences in intraindividual change. However, as we all develop a better understanding of these complex models of change, it is increasingly apparent that one of the most important advantages may well be the ability to disaggregate the between- and within-person influences of a TVC on the outcome variable. Indeed, such a disaggregation is of primary interest to most theories of human behavior (e.g., Curran & Bauer, 2011; Molenaar, 2004; Molenaar & Newell, 2011). Yet, at least in our opinion, insufficient attention has been paid to this disaggregation of effects both within the quantitative and substantive disciplines in the behavioral sciences. We hope that our work here has not only provided some unique insight into how the disaggregation of within- and between-person effects are manifested within the MLM and the SEM but has also chartered several directions in which future developments might proceed. These ongoing quantitative developments can in turn goose us to further refine our substantive theories so that we may better articulate precisely what type of change we posit when making predictions about trajectories of individual stability and change over time.

APPENDIX

Consider a true data generating model for a random intercept and slope with a single time varying covariate; the measurement and structural equations are

$$y_{ti} = \beta_{0i} + \beta_{1i} time_{ti} + \omega(z_{ti} - \bar{z}_i) + \varepsilon_{ti} \quad (9.A1)$$

$$\beta_{0i} = \mu_{\beta} + (\omega + \delta_0)\bar{z}_i + \zeta_{\alpha i} \quad (9.A2)$$

$$\beta_{1i} = \mu_{\beta} + (\omega + \delta_1)\bar{z}_i + \zeta_{\beta i} \quad (9.A3)$$

where ω is the within-person effect, $\omega + \delta_0$ is the between-person effect on the intercept, and $\omega + \delta_1$ is the between-person effect on the slope and all else is defined in the manuscript. The reduced form expression of this population model is thus

$$y_{ti} = \mu_\alpha + \mu_\beta^* time_{ti} + \zeta_{\alpha i}^* + \zeta_{\beta i}^* + \omega z_{ti} + \varepsilon_{ti} \quad (9.A4)$$

where $\mu_\beta^* = \mu_\beta + \omega\bar{z}_i$, $\zeta_{\alpha i}^* = \delta_0\bar{z}_i + \zeta_{\alpha i}$, and $\zeta_{\beta i}^* = \delta_1\bar{z}_i + \zeta_{\beta i}$.

Importantly, notice that

$$\text{var}(\zeta_{\alpha i}^*) = \delta_0^2 \text{var}(\bar{z}_i) + \psi_{\alpha\alpha} \quad (9.A5)$$

$$\text{var}(\zeta_{\beta i}^*) = \delta_1^2 \text{var}(\bar{z}_i) + \psi_{\beta\beta} \quad (9.A6)$$

$$\text{cov}(\zeta_{\alpha i}^*, \zeta_{\beta i}^*) = \delta_0\delta_1 \text{var}(\bar{z}_i) + \psi_{\alpha,\beta} \quad (9.A7)$$

$$\text{cov}(\zeta_{\alpha i}^*, \bar{z}_i) = \delta_0 \text{var}(\bar{z}_i) \neq 0 \quad (9.A8)$$

$$\text{cov}(\zeta_{\beta i}^*, \bar{z}_i) = \delta_1 \text{var}(\bar{z}_i) \neq 0 \quad (9.A9)$$

where $\psi_{\alpha\alpha} = \text{var}(\zeta_{\alpha i})$, $\psi_{\beta\beta} = \text{var}(\zeta_{\beta i})$, and $\psi_{\alpha\beta} = \text{cov}(\zeta_{\alpha i}, \zeta_{\beta i})$. Based on this information, we can derive estimates of δ_0 and δ_1 that in turn allows us to obtain direct estimates of the between-person effects. Specifically, δ_0 can be directly obtained by

$$\delta_0 = \frac{\text{cov}(\bar{z}_i, \zeta_{\alpha i}^*)}{\text{var}(\bar{z}_i)} \quad (9.A10)$$

$$= \frac{\frac{1}{T}\sum_{t=1}^{T}\text{cov}(z_{ti}, \zeta_{\alpha i}^*)}{\text{var}\left(\frac{1}{T}\sum_{t=1}^{T} z_{ti}\right)}$$

$$= \frac{\frac{1}{T}\sum_{t=1}^{T}\text{cov}(z_{ti}, \zeta_{\alpha i}^*)}{\frac{1}{T^2}\left\{\sum_{t=1}^{T}\text{var}(z_{ti}) + \sum_{t\neq t'}^{T}\text{cov}(z_{ti}, z_{t'i})\right\}}$$

Following similar logic δ_1 can be computed as

$$\delta_1 = \frac{\frac{1}{T}\sum_{t=1}^{T}\text{cov}(z_{ti}, \zeta_{\beta i}^*)}{\frac{1}{T^2}\left\{\sum_{t=1}^{T}\text{var}(z_{ti}) + \sum_{t\neq t'}^{T}\text{cov}(z_{ti}, z_{t'i})\right\}} \quad (9.A11)$$

As such, sample estimates of both of the contextual effects (i.e., $\hat{\delta}_0$ and $\hat{\delta}_1$) can be calculated using SEM results in equations 9.A10 and 9.A11. Direct estimates for the between-person effects can then be obtained by calculating $\hat{\omega} + \hat{\delta}_0$ and $\hat{\omega} + \hat{\delta}_1$. Standard delta method procedures can be used to obtain an associated standard error for this compound parameter estimate.

To stress, we do not anticipate that the calculation of $\hat{\delta}_0$ and $\hat{\delta}_1$ are likely of use in practice; we show these to explicate precisely how the between- and within-person effects can be analytically obtained from the SEM. We recommend that the latent variable approach for the TVCs be used for disaggregating effects in substantive applications.

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REFERENCES

Aiken, L. S., & West, S. G. (1991). *Multiple regression: Testing and interpreting interactions.* Newbury Park, CA: Sage.

Bauer, D. J. (2003). Estimating multilevel linear models as structural equation models. *Journal of Educational and Behavioral Statistics, 28,* 135–167.

Bollen, K. A. (1989). *Structural equations with latent variables.* New York: Wiley.

Bollen, K. A., & Curran, P. J. (2006). *Latent curve models: A structural equation approach.* Hoboken, NJ: Wiley.

Browne, M. W. (1984). Asymptotic distribution free methods in the analysis of covariance structures. *British Journal of Mathematical and Statistical Psychology, 37,* 127–141.

Bryk, A. S., & Raudenbush, S. W. (1987). Application of hierarchical linear models to assessing change. *Psychological Bulletin. 101,* 147–158.

Cattell, R. B. (1944). Psychological measurement: Normative, ipsative, interactive. *Psychological Review, 51,* 292–303.

Chan, W., & Bentler, P. M. (1996). Covariance structure analysis of partially additive ipsative data using restricted maximum likelihood estimation. *Multivariate Behavioral Research, 31,* 289–312.

Chassin, L., Curran, P. J., Hussong, A. M., & Colder, C. R. (1996). The relation of parent alcoholism to adolescent substance use: A longitudinal follow-up study. *Journal of Abnormal Psychology, 105,* 70–80.

Clemans, W. V. (1966). *An analytical and empirical examination of some properties of ipsative measures* (Psychometric Monograph No. 14). Richmond, VA: Psychometric Society. Retrieved from www.psychometrika.org/journal/online/MN14.pdf.

Cohen, J., Cohen, P., West, S., & Aiken, L. (2003). *Applied multiple regression/correlation analysis for the behavioral sciences (3rd ed.).* Hillsdale, NJ: Erlbaum.

Curran, P. J. (2003). Have multilevel models been structural equation models all along? *Multivariate Behavioral Research, 38,* 529–569.

Curran, P. J., & Bauer, D. J. (2011). The disaggregation of within-person and between-person effects in longitudinal models of change. *Annual Review of Psychology, 62,* 583–619.

Curran, P. J., & Bollen, K. A. (2001). The best of both worlds: Combining autoregressive and latent curve models. In L. M. Collins & A. G. Sayer (Eds.), *New methods for the analysis of change* (pp. 105–136). Washington, DC: American Psychological Association.

Dunlap, W. P., & Cornwell, J. J. (1994). Factor analysis of ipsative measures. *Multivariate Behavioral Research, 29,* 115–126.

du Toit, S. H. C., & Browne, M. W. (2001) *The covariance structure of a vector time series.* In R. Cudeck, S. H. C. du Toit, & D. Sörbom (Eds.), *Structural equation modeling: Present and future* (pp. 279–314). Chicago: Scientific Software International.

Flora, D. B., & Curran, P. J. (2004). An empirical evaluation of alternative methods of estimation for confirmatory factor analysis with ordinal data. *Psychological Methods, 9,* 466–491.

Greene, W. H. (2000). *Econometric analysis* (4th ed.). Upper Saddle River, NJ: Prentice Hall.

Hoffman, L., & Stawski, R. (2009). Persons as contexts: Evaluating between-person and within-person effects in longitudinal analysis. *Research in Human Development, 6,* 97–100.

Hofmann, D. A., & Gavin, M. B. (1998). Centering decisions in hierarchical linear models: Theoretical and methodological implications for organizational science. *Journal of Management, 24,* 623–641.

Kaplow, J. B., Curran, P. J., & Costello, E. J. (2001). The prospective relation between dimensions of anxiety and the initiation of adolescent alcohol use. *Journal of Clinical Child Psychology, 30,* 316–326.

Kassel, D. J., Hussong, A. M., Wardle, M. C., Veilleux, J. C., Heinz, A., Greenstein, J. E., et al. (2010). *Affective influences in drug use etiology.* In L. M. Scheier (Ed.), *Handbook of drug use etiology: Theory, methods and empirical findings* (pp. 183–206). Washington, DC: American Psychological Association.

Kreft, I. G. G., de Leeuw, J., & Aiken, L. S. (1995). The effect of different forms of centering in hierarchical linear models. *Multivariate Behavioral Research, 30,* 1–21.

Lüdtke, O., Marsh, H. W., Robitzsch, A. Trautwein, U., Asparouhov, T., & Muthén, B. (2008). The multilevel latent covariate model: A new, more reliable approach to group-level effects in contextual studies. *Psychological Methods, 13,* 203–229.

MacCallum, R. C., Kim, C., Malarkey, W. B., & Kiecolt-Glaser, J. K. (1997). Studying multivariate change using multilevel models and latent curve models. *Multivariate Behavioral Research, 32,* 215–253.

McArdle, J. J. (1988). Dynamic but structural equation modeling of repeated measures data. In J. R. Nesselroade & R. B. Cattell (Eds.), *Handbook of multivariate experimental psychology* (2nd ed.). New York: Plenum Press.

McArdle, J. J. (1989). Structural modeling experiments using multiple growth functions. In P. Ackerman, R. Kanfer, & R. Cudeck (Eds.), *Learning and individual differences: Abilities, motivation and methodology* (pp. 71–117). Hillsdale, NJ: Erlbaum.

McArdle, J. J. (1991). Structural models of developmental theory in psychology. In P. Van Geert & L. P. Mos (Eds.), *Annals of theoretical psychology, Volume VII* (pp. 139–160). New York: Plenum Press.

McArdle, J. J., & Epstein, D. (1987). Latent growth curves within developmental structural equation models. *Child Development, 58,* 110–133.

McArdle, J. J., & Hamagami, F. (2001). Latent difference score structural models for linear dynamic analyses with incomplete longitudinal data. In L. M. Collins & A. G. Sayer (Eds.), *New methods for the analysis of change* (pp. 137–175). Washington, DC: American Psychological Association.

Mehta, P. D., & Neale, M. C. (2005). People are variables too: Multilevel structural equations modeling. *Psychological Methods, 10,* 259–284.

Mehta, P. D., Neale, M. C., & Flay, B. R. (2004). Squeezing interval change from ordinal panel data: Latent growth curves with ordinal outcomes. *Psychological Methods, 9,* 301–333.

Meredith, W., & Tisak, J. (1984). *"Tuckerizing" curves.* Paper presented at the annual meeting of the Psychometric Society, Santa Barbara, CA.

Meredith, W., & Tisak, J. (1990). Latent curve analysis, *Psychometrika, 55,* 107–122.

Molenaar, P. C. M. (2004). A manifesto on psychology as idiographic science: Bringing the person back into scientific psychology, this time forever. *Measurement: Interdisciplinary Research and Perspective, 2,* 201–218.

Molenaar, P. C. M., & Newell, K. M. (2011). *Individual pathways of change: Statistical models for analyzing learning and development.* Washington, DC: American Psychological Association.

Raudenbush, S. W. (2001). Toward a coherent framework for comparing trajectories of individual change. In L. M. Collins & A. G. Sayer (Eds.), *New methods for the analysis of change* (pp. 35–64). Washington, DC: American Psychological Association.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models* (2nd ed.). Thousand Oaks, CA: Sage.

Rodebaugh, T., Curran, P., & Chambless, D. (2002). Expectancy of panic in the maintenance of daily anxiety in panic disorder with agoraphobia: A longitudinal test of competing models. *Behavior Therapy, 33,* 315–336.

Rogosa, D. R., & Saner, H. M. (1995). Longitudinal data analysis examples with random coefficient models. *Journal of Educational and Behavioral Statistics, 20,* 149–170.

Satorra, A. (1990). Robustness issues in structural equation modeling: A review of recent developments. *Quality & Quantity, 24,* 367–386.

Schwartz, J. E., & Stone, A. A. (1998). Strategies for analyzing ecological momentary assessment data. *Health Psychology, 17,* 6–16.

Singer, J. D., & Willett, J. B. (2003). *Applied longitudinal data analysis: Modeling change and event occurrence.* New York: Oxford University Press.

Willett, J. B., & Sayer, A. G. (1994). Using covariance structure analysis to detect correlates and predictors of individual change over time. *Psychological Bulletin, 116,* 363–381.

NOTES

1. It is possible under some conditions to estimate random effects for the TVCs within the SEM using definition variables (e.g., Mehta & Neale, 2005), but this is moving well beyond our focus here.
2. To maintain maximum generality we retain the cross-level interaction between the person-specific mean and the measure of time, although in our later simulations we fix this value to zero in order to unambiguously define the main effect of the person-specific mean as the between effect.
3. Formally testing this hypothesis would make an excellent dissertation.
4. All population parameter values are presented in the final column of Table 9.2 using SEM notation that is defined later in the chapter.
5. For simplification, we make the unrealistic assumption of no missing data. However, this imposes no limitations on the developments we present here. Indeed, we reestimated all models with 10% and 20% missing data under MAR mechanisms, and as expected all key conclusions remained unchanged.
6. We include an estimate of the cross-level interaction between the person-specific mean and time, although the value of this effect was fixed to zero in the population.
7. Within the SEM the TVC can take on unique values across time by freely estimating γ at each time point t; in the MLM the same effect can be obtained by entering an interaction between the TVC and the measure of time.
8. It is also possible to grand mean center the TVCs within the SEM, and this strategy offers certain interpretational advantages with respect to the latent variable means. However, grand mean centering does not allow us to obtain direct estimates of the within- and between-person effects of the TVC on the outcome, so we do not pursue this further here.
9. Note that in some cases it might be possible to estimate two separate models, one that contains just the TVC and one that contains just the person-means. The estimates of the within- and between-person effects can then be obtained from each. However, this approach is statistically inefficient, is prone to adverse effects of model misspecification, and limits the estimation of more general models.
10. We can also define a compositional effect for the linear slope component, but here we focus on the more common main effect of the between effect of the TVC predicting the random intercept; see the Appendix for a derivation of the compositional effect for the slope component as well.
11. Here we are assuming that T is constant for all individuals in the sample, but the equations can be adjusted accordingly for unbalanced designs.

12. When the latent factor for the TVC is allowed to covary with that of the outcome (as it is here), the within-person effect is equivalently captured by regressing the repeated measures on either the residual of the TVC or directly on the TVC itself. However, regressing the repeated measures on the residuals is a more general strategy that allows for the estimation of alternative models that we do not detail here.
13. The multivariate equations for this model can be written, but these expressions become tedious in scalar form and requires matrix notation. Given space constraints we do not pursue this further here; see Bollen and Curran (2006, pp. 188–207) for details about these multivariate expressions.
14. It is currently unknown the extent to which these models might be estimable using direct ML estimation that is not based on summary statistics as the unit of analysis; more work is needed to better understand this possibility.